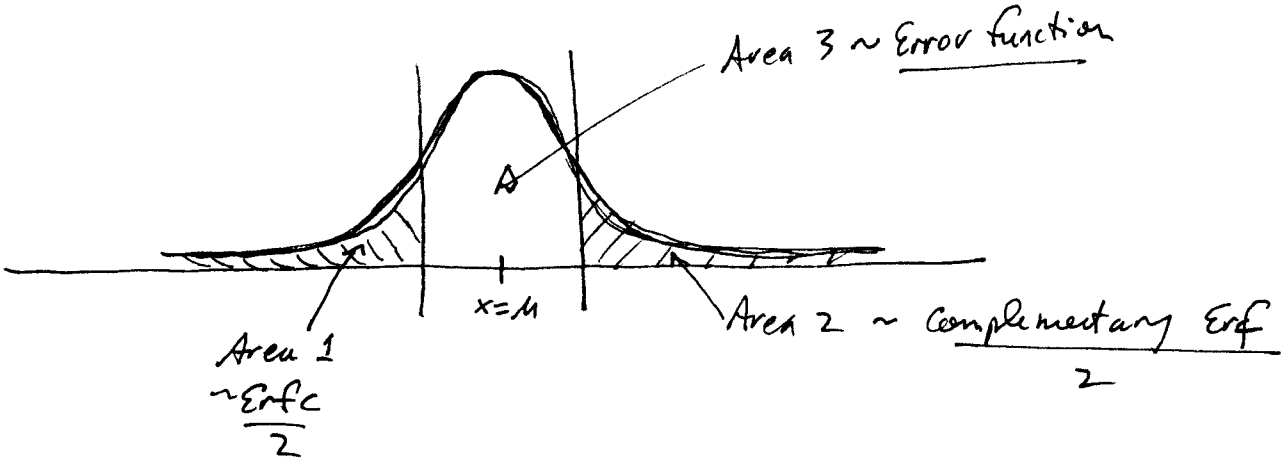
1/24/05 – Extracting noise from a threshold curve:

Efficiency = Prob that (input + noise) > threshold

Input + noise = Gaussian centered @ Input pulse ht

$$P_G(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2}$$



For $x \le \mu$, Efficiency $= \frac{Erfc}{2}$

for $x \ge \mu$, Efficiency $= 1 - \frac{Erfc}{2}$

Use $z = (x-\mu)/\sigma$ $\Rightarrow$ Integrate a Gaussian centered at $z=0$, with $\sigma = 1$

use $\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

$$P_G(x, \mu=0, \sigma=1) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x}{\sqrt{2}}\right)^2}$$

$$\int_{-y}^{y} P_G(x,0,1)\,dx = \frac{1}{\sqrt{2\pi}} \int_{-y}^{y} e^{-\left(\frac{x}{\sqrt{2}}\right)^2} dx = \frac{1}{\sqrt{2\pi}} \int_{-y/\sqrt{2}}^{y/\sqrt{2}} e^{-t^2} \sqrt{2}\, dt$$

$\Rightarrow \cancel{\int_{-y}^{y} P_G\, dx =}$

this step uses the substitution $x = \sqrt{2}\,t$; $dx = \sqrt{2}\,dt$

($\Rightarrow$ $x = y$ is $t = y/\sqrt{2}$)

$$\int_{-y}^{y} P_G(x,0,1)\,dx = \frac{1}{\sqrt{\pi}} \int_{-y/\sqrt{2}}^{y/\sqrt{2}} e^{-t^2}\,dt$$

& e^{-t^2} is symmetric about $t=0$, so

$$\int_{-y}^{y} P_G(x,0,1)\,dx = \frac{2}{\sqrt{\pi}} \int_{0}^{y/\sqrt{2}} e^{-t^2}\,dt = \mathrm{erf}(y/\sqrt{2})$$

$$\Rightarrow \int_{-\infty}^{y} P_G(x,0,1)\,dx = \frac{\mathrm{erfc}(y/\sqrt{2})}{2} \quad \text{for } y \le 0$$

&

$$\int_{-\infty}^{y} P_G(x,0,1)\,dx = 1 - \frac{\mathrm{erfc}(y/\sqrt{2})}{2} \quad \text{for } y \ge 0$$

$\Rightarrow$ Procedure for fitting Efficiency Curve (μ, σ):

Transform to $\mu = 0$, $\sigma = 1$ with $z = (x-\mu)/\sigma$, then use fitting function as above.

```
        subroutine nf_calculate(x, y, param, yp)
c*************************************************************************
c
c   This subroutine computes the integral from (-infinity) to z of
c    a normalized Gaussian centered at 0: P(zz)=1/sqrt(2Pi)*exp(-zz/sqrt(2))^2
c
c*************************************************************************
        implicit none

        double precision x, xLocal, y, param(*), yp
        double precision z, answer, s2
      double precision derfc
      external derfc

      data s2/1.414213562d0/
        z = (x - param(1) ) * param(2)
      if(z.le.0.)goto 10
c-->z>0
      xLocal=z/s2
      answer=1.- derfc(xLocal)/2.
      goto 50
10    continue
c-->z.le.0
      xLocal=-z/s2
      answer = derfc(xLocal)/2.
50    continue
        yP = answer
c     print *, param(1), param(2)
c     write(6,2) x,yP
2     format(1x,4e16.8)
         return
         end
```

param(1) = mean

param(2) = scale = $1/\sigma$

Note: y isn't used... it's left over from some previous incarnation of the fitting program!